Shear viscosity of pion gas

Eiji Nakano

Department of Physics and Center for Theoretical Sciences,
National Taiwan University, Taipei 10617,Taiwan.
enakano@ntu.edu.tw

1 Introduction

Shear viscosity η is one of transport coefficients in fluid dynamics, which characterizes how viscous the system is in the presence of flow gradient [1]. Since, in general, shear viscosity is inversely proportional to scattering cross-section, strongly interacting systems have smaller viscosity than weakly interacting ones. Recently a universal minimum bound for the ratio of η to entropy density s has been proposed by Kovtun, Son, and Starinets (KSS) [2]. The bound, $\eta/s \geq \frac{1}{4\pi}$, is conjectured to be satisfied for a large class of strongly interacting quantum field theories whose dual descriptions in string theory involve black holes in anti-de Sitter space [3, 4, 5, 6]. Note that η/s is more physical quantity than η itself because the ratio appears as a diffusion constant of fluid equations.

In experiments, η/s close to the minimum bound were found in relativistic heavy ion collisions (RHIC) [7, 8, 9]. This discovery came as a surprise. Traditionally, quark gluon plasma (QGP), a phase of the quantum chromodynamics (QCD) above the deconfinement temperature $T_c \sim 170$ MeV at zero baryon density [10], had been thought to be in weak interaction regime. Partly because lattice QCD simulations of the QGP equation of state above $2T_c$ are not inconsistent with that of an ideal gas of massless particles, $e = 3p$, where e is the the energy density and p is the pressure of the system [10]. However, recent analyses of the elliptic flow generated by non-central collisions in RHIC [8, 9] and lattice simulations of a gluon plasma [11] yielded η/s close to the the minimum bound at just above T_c. This suggests QGP is strongly interacting at this temperature. [1] Other implications for strong coupling can be seen in sharp peaks of mesonic correlators [12, 13, 14], and in discussions of the possible microscopic structure of such a state [16, 17, 18, 19, 20].

Given this situation, one naturally gets interested in how η/s behaves as temperature approaches T_c from below, supposing that η/s of QCD was

[1] See Ref. [15] for other possibility to yield a small viscosity in expanding QGP.

already close to the minimum bound at just above T_c, and how it relates to change of the effective degrees of freedom through a phase transition or cross over.

To explore these issues, we use chiral perturbation theory (ChPT) and the linearized Boltzmann equation to perform a model independent calculation to the η/s of QCD in the confinement phase. Earlier attempts to compute meson matter viscosity using the Boltzmann equation and phenomenological phase shifts in the context of RHIC hydrodynamical evolution after freeze out can be found in Refs. [26, 27, 28]. In the deconfinement phase, state of the art perturbative QCD calculations of η can be found in Refs. [29, 30].

1.1 Shear viscosity in Fluid dynamics

For later understanding it might be worth mentioning basic properties of shear viscosity. Fluid dynamics describes non-equilibrium system where evolution in space and time occurs in macroscopic scale. η appears as a phenomenological parameter in this scale. Usually we can select suitable theory corresponding to the characteristic evolution scale of our interest, as listed in table 1.

Table 1. Hierarchy of theories in space-time scales

Scale	Theory
Micro (range of interaction)	Quantum theory, e.g., Linear response theory
Meso (mean-free path)	Kinetic theory, e.g., Boltzmann eq.
Macro (sound wave)	Fluid dynamics, e.g., Navier-Stokes eq. with η

Fluid dynamics in relativistic framework is consist of two basic equations, that is, conservation laws of the energy-momentum $T^{\mu\nu}(x)$ and the number density (or charge) $n(x)$:

$$\partial_\mu T^{\mu\nu}(x) = 0, \tag{1}$$

$$\partial_\mu n(x) V^\mu(x) = 0, \tag{2}$$

where $V^\mu(x)$ is a normalized four velocity of elementary volume, $V^\mu(x)V_\mu(x) = 1$. In what follows, we will not care the second equation in treating pion gas, because pion does not carry any conserved charge. The energy-momentum tensor in viscous system is decomposed as

$$T_{\mu\nu}(x) = T^{(0)}_{\mu\nu}(x) + \delta T_{ij}(x), \tag{3}$$

where the first term describes the perfect fluid (local equilibrium; frictionless processes) with local pressure $\mathcal{P}(x)$ and energy density $\epsilon(x)$

$$T^{(0)}_{\mu\nu}(x) = \{\mathcal{P}(x) + \epsilon(x)\}\, V_\mu(x)V_\nu(x) - \mathcal{P}(x)\delta_{\mu\nu}, \tag{4}$$

and the second term corresponds to a small deviation from local equilibrium, which defines shear and bulk viscosities, η and ζ,

$$\delta T_{ij}(x) = -\eta\left(\nabla_i V_j(x) + \nabla_j V_i(x) - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{V}(x)\right) + \zeta\delta_{ij}\nabla\cdot\mathbf{V}(x). \quad (5)$$

One can see that η (ζ) is in traceless (trace) part of spatial components. These transport coefficients, which also includes thermal conductivity in the presence of conserved charge, are determined by experiments, or can be derived from more microscopic theories in principle. In general, it becomes more difficult to derive them as one begins with smaller scale.

We shall estimate η by considering two dimensional case depicted in Fig. 1, where x-component of flow velocity $V_x(y)$ varies along y direction. In other

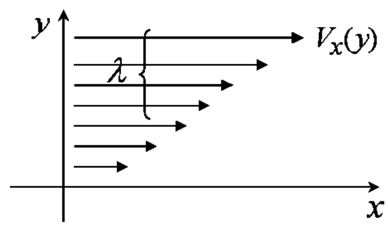

Fig. 1. Shear flow on x-y plane.

words, shear viscosity is an anisotropic pressure defined by the momentum transfer of x-component per unit time across unit area normal to y direction, in which the force is facing negative x direction. Since the momentum transfer occurs within the mean-free path λ,

$$\delta T^{\mu\nu} = -\eta\frac{\partial V_x(y)}{\partial y} \simeq -nV_y\frac{\partial p_x}{\partial y}\lambda = -\frac{p_y}{\sigma}\frac{\partial V_x(y)}{\partial y}, \quad (6)$$

where we have used $\lambda = 1/(n\sigma)$ with σ and n being scattering cross-section and density. Thus, we can roughly estimate it by dimensional analysis up to a numerical factor,

$$\eta \simeq \frac{T}{\sigma} \simeq \frac{T^3}{|\mathcal{T}|^2}, \quad (7)$$

where we have reduced momentum to temperature $p \sim T$, and $\mathcal{T}$ is the scattering amplitude. It shows that strong interaction (small λ) causes quick and local equilibrium, implying that the system can be well described by hydrodynamics. In the case of weak interaction (large λ), on the other hand, a large number of particles (thermal excitations) included in λ join to relax the gradient of flow velocity, so that, it makes η finite.

1.2 Shear viscosity in Kubo formula

If one would like to evaluate η in field theoretical way with Lagrangian, the linear response theory gives a formulation. Since η is a response of the system to the external stimulation δT_{ij}, it is given by Kubo formula

$$\eta = -\frac{1}{5}\int_{-\infty}^{0} \mathrm{d}t' \int_{-\infty}^{t'} \mathrm{d}t \int \mathrm{d}x^3 \langle \left[T^{ij}(0), T^{ij}(\mathbf{x}, t)\right] \rangle, \tag{8}$$

where T^{ij} the spatial off-diagonal part of energy-momentum tensor, and is operator derived from Lagrangian of the system. This formula gives the strict definition of η derived from microscopic Lagrangian. One might think a perturbative calculation of the above correlator will give an answer for η. But this is not true even for weak coupling theory. Indeed, the Kubo formula involves, for instance in $\lambda\phi^4$ theory, an infinite number of diagrams at the leading order (LO) [24]. This is due to an infrared singularity (remember that η appears in macroscopic scale). However, it is proven that the summation of LO diagrams is equivalent to solving the linearized Boltzmann equation (for mesoscopic scale) with temperature dependent particle masses and scattering amplitudes [24]. The result shows $\eta \sim T^3/\lambda^2$. This proof extended the applicable range of the Boltzmann equation to higher temperature, but is restricted to weak coupling theories. In the case we are interested (QCD with $T < 140$ MeV), the pion mean free path is always greater than the range of interaction (~ 1 fm) by a factor of 10^3. Thus, even though the coupling in ChPT is too strong to use the result of $\lambda\phi^4$ theory in [24], the temperature is still low enough that the use of the Boltzmann equation is justified. In the following sections, we will show shear viscosity of pion gas evaluated from the linearized Boltzmann equation to LO in ChPT, and develop arguments on results, which is all based on our recent paper Ref. [25].

2 Linearized Boltzmann Equation for Low Energy QCD

In the hadronic phase of QCD with zero baryon-number density, the dominant degrees of freedom are the lightest hadrons—the pions. The pion mass $m_\pi = 139$ MeV is much lighter than the mass of the next lightest hadron—the kaon whose mass is 495 MeV. Given that T_c is only ~ 170 MeV, it is sufficient to just consider the pions in the calculation of thermodynamical quantities and transport coefficients for $T \ll T_c$.

The interaction between pions can be described by chiral perturbation theory (ChPT) in a systematic expansion in energy and quark (u and d quark) masses [21, 22, 23]. ChPT is a low energy effective field theory of QCD. It describes pions as Nambu-Goldstone bosons of the spontaneously broken chiral symmetry. At $T \ll T_c$, the temperature dependence in $\pi\pi$ scattering can be calculated systematically. At $T = T_c$, however, the theory breaks down due to the restoration of chiral symmetry.[2]

Since unlike the Kubo formula the Boltzmann equation requires semi-classical descriptions of particles with definite position, energy and momentum except during brief collisions, the mean-free path is required to be much greater than the range of interaction. Thus the Boltzmann equation is usually limited to low temperature systems. The Boltzmann equation describes the evolution of the isospin averaged pion distribution function $f = f(\mathbf{x}, \mathbf{p}, t) \equiv f_p(x)$ (a function of space, time and momentum) as

$$\frac{p^\mu}{E_p}\partial_\mu f_p(x) = \frac{g_\pi}{2}\int_{123} d\Gamma_{12;3p}\left\{f_1 f_2(1+f_3)(1+f_p) - (1+f_1)(1+f_2)f_3 f_p\right\} \tag{9}$$

where $E_p = \sqrt{\mathbf{p}^2 + m_\pi^2}$ and $g_\pi = 3$ is the degeneracy factor for three pions,

$$d\Gamma_{12;3p} \equiv \frac{1}{2E_p}|\mathcal{T}|^2 \prod_{i=1}^{3} \frac{d^3\mathbf{k}_i}{(2\pi)^3(2E_i)} \times (2\pi)^4\delta^4(k_1 + k_2 - k_3 - p)\ , \tag{10}$$

and where $\mathcal{T}$ is the scattering amplitude for particles with momenta $1, 2 \to 3, p$. LHS of (9) gives temporal change of $f_p(x)$, and it should be equal to change by collision (RHS) in which the first (second) term corresponds to gain (loss) of particles of momentum p by the collision.

In ChPT, the LO isospin averaged $\pi\pi$ scattering amplitude in terms of Mandelstam variables (s, t, and u) is given by

$$|\mathcal{T}|^2 = \frac{1}{9}\sum_{I=0,1,2}(2I+1)|\mathcal{T}^{(I)}|^2 = \frac{1}{9f_\pi^4}\left\{21m_\pi^4 + 9s^2 - 24m_\pi^2 s + 3(t-u)^2\right\}\ . \tag{11}$$

The temperature dependence in pion mass and pion scattering amplitudes can be treated as higher order corrections.

The distribution function can be decomposed into the local equilibrium $f_p^{(0)}(x)$ and a deviation from it,

$$f_p(x) = f_p^{(0)}(x) + \delta f_p(x), \tag{12}$$

where $f_p^{(0)}(x) = \left(e^{\beta(x)V_\mu(x)p^\mu} - 1\right)^{-1}$ with $\beta(x)$ being the inverse temperature and $V^\mu(x)$ the four velocity at the space-time point x. A small deviation $\delta f_p(x)$ from the local equilibrium is parameterized as

$$f_p(x) = f_p^{(0)}(x)\left[1 - \left\{1 + f_p^{(0)}(x)\right\}\chi_p(x)\right], \tag{13}$$

while the energy momentum tensor is

[2] The QCD chiral restoration happens almost as soon as the deconfinement transition at zero baryon density. We do not distinguish their critical temperatures.

$$T_{\mu\nu}(x) = T^{(0)}_{\mu\nu}(x) + \delta T_{\mu\nu}(x) = g_\pi \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{p_\mu p_\nu}{E_p} \left[f^{(0)}_p(x) + \delta f_p(x) \right]. \quad (14)$$

We will choose the $\mathbf{V}(x) = 0$ frame for the point x. This implies $\partial_\nu V^0 = 0$ after taking a derivative on $V_\mu(x)V^\mu(x) = 1$. Furthermore, the conservation law at equilibrium $\partial_\mu T^{\mu\nu}|_{\chi_p=0} = 0$ allows us to replace $\partial_t \beta(x)$ and $\partial_t \mathbf{V}(x)$ by terms proportional to $\nabla \cdot \mathbf{V}(x)$ and $\nabla\beta(x)$. Thus, to the first order in a derivative expansion, $\chi_p(x)$ can be parameterized as

$$\frac{\chi_p(x)}{\beta(x)} = A(p)\nabla \cdot \mathbf{V}(x) + B_{ij}(p) \left(\frac{\nabla_i V_j(x) + \nabla_j V_i(x)}{2} - \delta_{ij} \frac{\nabla \cdot \mathbf{V}(x)}{3} \right), \quad (15)$$

where $B_{ij}(p) \equiv B(p) \left(\hat{p}_i \hat{p}_j - \frac{1}{3}\delta_{ij} \right)$, and i and j are spatial indexes. [3]

Substituting (15) into the Boltzmann equation, one obtains a linearized equation for $B(p)$,

$$\begin{aligned} \left(p_i p_j - \frac{1}{3}\delta_{ij}\mathbf{p}^2 \right) &= \frac{g_\pi E_p}{2} \int_{123} d\Gamma_{12;3p} (1+n_1)(1+n_2)n_3(1+n_p)^{-1} \\ &\quad \times \left[B_{ij}(p) + B_{ij}(k_3) - B_{ij}(k_2) - B_{ij}(k_1) \right] \\ &\equiv g_\pi \hat{F}_{ij}\left[B \right], \end{aligned} \quad (16)$$

where we have dropped the factor $(\nabla_i V_j(x) + \nabla_j V_i(x) - \text{trace})$ contracting both sides of the equation and write $f^{(0)}_i(x)$ at this point as $n_i = \left(e^{\beta E_i} - 1 \right)^{-1}$. There is another integral equation associated with $\nabla \cdot \mathbf{V}(x)$ which is related to the bulk viscosity ζ that will not be discussed here. The $\nabla \cdot \beta$ and $\partial_t \mathbf{V}$ terms in $p^\mu \partial_\mu f^{(0)}_p$ will cancel each other by the energy momentum conservation in local equilibrium mentioned above.

After putting everything together and comparing the energy-momentum tensor (5) in fluid dynamics and (14) in kinetic theory, we obtain

$$\eta = \frac{g_\pi \beta}{10} \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{1}{E_p} n_p \left(1+n_p \right) B_{ij}(p) \left(p_i p_j - \frac{1}{3}\delta_{ij}\mathbf{p}^2 \right) \quad (17)$$

$$= \frac{g^2_\pi \beta}{10} \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{1}{E_p} n_p \left(1+n_p \right) B_{ij}(p) \hat{F}_{ij}\left[B \right] \equiv g^2_\pi \langle B|\hat{F}[B] \rangle \ . \quad (18)$$

Here one can see immediately that if the scattering cross section is scaled by a factor α,

$$d\Gamma_{12;3p} \to \alpha \left(d\Gamma_{12;3p} \right) \ , \quad (19)$$

then Eqs. (16) and (18) imply the following scaling

[3] A non-derivative term is not allowed since f_p should be reduced to $f^{(0)}_p$ when β and V^μ become independent of x. There is no term with single spatial derivative on $\beta(x)$ either. The only possible term $(\mathbf{V} \cdot \nabla)\,\beta(x)$ vanishes in the $\mathbf{V}(x) = 0$ frame.

$$B_{ij}(p) \to \alpha^{-1} B_{ij}(p) \ , \tag{20}$$

$$\eta \to \alpha^{-1} \eta \ , \tag{21}$$

with η proportional to the inverse of scattering cross-section. This non-perturbative result is a general feature for the linearized Boltzmann equation with two-body elastic scattering.

To find a solution for $B(p)$, one can just solve Eq. (16). But here we follow the approach outlined in Refs. [27, 28] to assume that $B(p)$ is a smooth function which can be expanded using a specific set of orthogonal polynomials

$$B(p) = |\mathbf{p}|^y \sum_{r=0}^{\infty} b_r B^{(r)}(z(p)), \tag{22}$$

where $B^{(r)}(z)$ is a polynomial up to z^r and b_r is its coefficient. The overall factor $|\mathbf{p}|^y$ will be chosen by trial and error to get the fastest convergence. The orthogonality condition

$$\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{\mathbf{p}^2}{E_p} n_p \left(1 + n_p\right) |\mathbf{p}|^y B^{(r)}(z) B^{(s)}(z) \propto \delta_{r,s} \tag{23}$$

can be used to construct the $B^{(r)}(z)$ polynomials up to normalization constants. For simplicity, we will choose $B^{(0)}(z) = 1$.

With this expansion, the consistency condition for $B(p)$ in Eq. (18) yields

$$\eta = g_\pi \sum_r b_r L^{(r)} = g_\pi^2 \sum_{r,s} b_r \langle B^{(r)} | \hat{F}[B^{(s)}] \rangle b_s \ , \tag{24}$$

where

$$L^{(r)} = \frac{\beta}{15} \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{\mathbf{p}^2}{E_p} n_p \left(1 + n_p\right) |\mathbf{p}|^y B^{(r)}(p) \propto \delta_{0,r}. \tag{25}$$

Since b_r is a function of m_π, f_π and T, the b_r's in Eq. (24) are in general independent functions, such that $L^{(r)} = g_\pi \sum_s \langle B^{(r)} | \hat{F}[B^{(s)}] \rangle b_s$ [one can show that this solution of b_s gives a unique solution of η], or equivalently

$$\delta_{0,r} L^{(0)} = g_\pi \sum_s \langle B^{(r)} | \hat{F}[B^{(s)}] \rangle b_s \ . \tag{26}$$

This will allow us to solve for the b_s and obtain the shear viscosity

$$\eta = g_\pi b_0 L^{(0)}. \tag{27}$$

In the next section, we will show that this expansion converges well, such that one does not need to keep many terms on the right hand side of Eq. (26). If only the $s = 0$ term was kept, then

$$\eta \simeq \frac{\left(L^{(0)}\right)^2}{\langle B^{(0)} | \hat{F}[B^{(0)}] \rangle}. \tag{28}$$

This resultant formula clearly shows that η does not depend on the degeneracy factor g_π.

The calculation of the entropy density s is more straightforward since s, unlike η, does not diverge in a free theory. In ChPT, the interaction contributions are all higher order in our LO calculation. Thus we just compute the s for a free pion gas:

$$s = -g_\pi \beta^2 \frac{\partial}{\partial\beta} \frac{\log Z}{\beta} \ , \tag{29}$$

where the partition function Z for free pions is

$$\frac{\log Z}{\beta} = -\frac{1}{\beta} \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \log\left\{1 - e^{-\beta E(p)}\right\} \ , \tag{30}$$

up to temperature independent terms.

3 Results and Discussion

3.1 Shear viscosity of Pion gas

We present the results for η and η/s of QCD at zero baryon number density. Fig. 2(a) shows the LO ChPT result of η using the linearized Boltzmann equation. For comparison, we also added results for a π-π scattering of empirical phase shifts (PS), in which values of parameters were employed from Refs. [26, 27, 28, 31]. Characteristically, the PS scattering amplitude does not reflect the phase transition so much to be less dependent on momentum than ChPT at high energy region, and incorporates resonance effect which is absent in ChPT. The lines with circles, squares and triangles correspond to keeping the first one, two and three polynomials on the right hand side of Eq. (26), respectively. We have used $y = 0$ and $z(p) = |\mathbf{p}|$ for ChPT and $y = 2$ and $z(p) = |\mathbf{p}|^2$ for PS to construct the polynomials. The figure shows the polynomial expansion converges rapidly.

As a test of the calculation, we also show the shear viscosity result for a constant scattering amplitude $|\mathcal{T}|^2 = \frac{23 m_\pi^4}{9 f_\pi^4}$ in Fig. 2(b) (ChPT SH), which corresponds to the low energy limit of ChPT and can be mapped onto $\lambda\phi^4$ theory of Ref. [24] where the scattering amplitude is a constant $|\mathcal{T}|^2 = \lambda^2$. In $\lambda\phi^4$ theory, η is monotonically increasing in T. If $T \gg m_\phi$, $\eta \propto T^3/\lambda^2$ with T^3 given by dimensional analysis and λ^{-2} by the scaling of coupling, as shown in Eq. (7) and in Eqs. (19) and (21). In ChPT, however, η is not monotonic in T. At $T \ll m_\pi$, the scattering amplitude is close to a constant, thus ChPT behaves like a $\lambda\phi^4$ theory. But as temperature gets higher to $T \gg m_\pi$, $\mathcal{T} \propto T^2/f_\pi^2$ and thus $\eta \propto f_\pi^4/T$. At what temperature the transition from $\eta \propto T^3$ to $\eta \propto 1/T$ takes place depends on the detail of dynamics. In ChPT, this temperature is around 20 MeV.

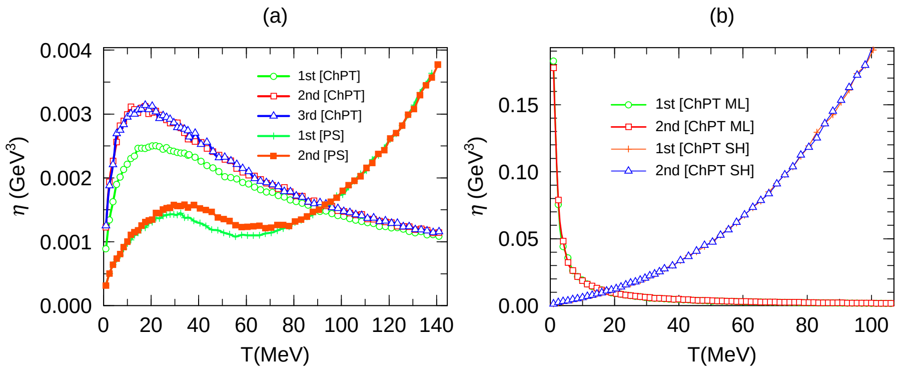

Fig. 2. Shear viscosity as a function of temperature. (a) for the LO ChPT (ChPT) and for an empirical phase-shift scattering amplitude (PS) [31]. $m_\pi = 139$ MeV and $f_\pi = 93$ MeV. (b) for the LO ChPT in the massless limit (ChPT ML) where $f_\pi = 87$ MeV, and for a constant scattering amplitude in low-energy limit, $|\mathcal{T}|^2 = \frac{23m_\pi^4}{9f_\pi^4}$ (ChPT SH). $1_{\rm st}$- $3_{\rm rd}$ show results up to the three polynomials in Eq. (26).

It is also interesting to observe the result of ChPT in massless limit, Fig. 2(b) (ChPT ML). η blows up at $T = 0$. It reflects the derivative coupling typical for NG modes, see Ref. [35] for η of phonon in CFL phase.

3.2 η/s and error estimation

The radius of convergence in momentum for ChPT is typically $4\pi f_\pi \sim 1$ GeV. To translate this radius into temperature, we compute the averaged center of mass momentum $\langle|\mathbf{p}|\rangle = \sqrt{\langle B|\mathbf{p}^2|\hat{F}[B]\rangle/\langle B|\hat{F}[B]\rangle}$. We found that for $T = 120$ and 140 MeV, $\langle|\mathbf{p}|\rangle \simeq 460$ and 530 MeV $< 4\pi f_\pi$. However, ChPT is supposed to break down at the chiral restoration temperature (~ 170 MeV). Thus our LO ChPT result can only be trusted up to $T \sim 120$ MeV. At the next-to-leading order (NLO), it is known that the isoscalar $\pi\pi$ scattering length will be increased by $\sim 40\%$ [23]. This will increase the cross section by $\sim 100\%$ and reduce η by $\sim 50\%$ near threshold. This is an unusually large NLO correction since a typical NLO correction at threshold is $\leq 20\%$. The large chiral corrections does not persist at the higher order. At the next-to-next-to-leading order (NNLO), the correction is $\sim 10\%$ [23]. Thus, to compute η to 10% accuracy, a NLO ChPT calculation is needed.

The LO ChPT result for η/s is shown in Fig. 3 (line with rectangles). The error is estimated to be $\sim 50\%$ up to 120 MeV. η/s is monotonically decreasing and reaches 0.6 at $T = 120$ MeV. This is similar to the behavior in the $m_\pi = 0$ case (shown as the line with rectangles) where $\eta/s \propto f_\pi^4/T^4$ with $s \propto T^3$ from dimensional analysis and $f_\pi = 87$ MeV in the chiral limit [22, 23]. For comparison, we also show the result using phenomenological π-π phase shifts [31] for η but free pions for s. (Our result for η is in good

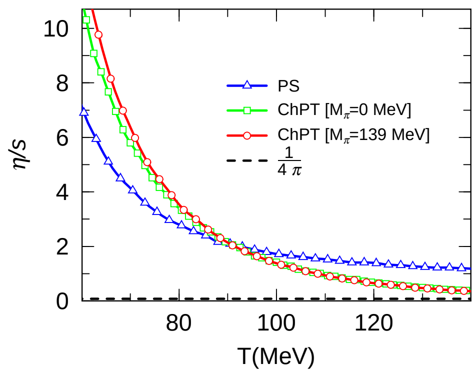

Fig. 3. Shear viscosity to entropy density ratios as functions of temperature. Line with circles (rectangles) is the LO ChPT result with $m_\pi = 139\,(0)$ MeV and $f_\pi = 93(87)$ MeV. Line with triangles is the result using $\pi\pi$ phase shifts (PS). Dashed line is the conjectured KSS bound $1/4\pi \simeq 0.08$.

agreement with that of [28] for T between 60 and 120 MeV. For an earlier calculation using the Chapman-Enskog approximation, see Ref. [32].) This amounts to taking into account part of the NLO $\pi\pi$ scattering effects but ignoring its temperature dependence and the interaction in s. Since not all the NLO effects are accounted for, this η/s is not necessarily more accurate than that using the LO ChPT. The comparison, however, gives us some feeling of the size of error for the LO result we present here. Thus, an error of $\sim 100\%$ at $T = 120$ MeV to the LO result might be more realistic.

3.3 Relation between T_c and η/s

Naive extrapolations of the three η/s curves show that the $1/4\pi = 0.08$ minimum bound conjectured from string theory might never be reached as in phase shift result [33, 34] (the first scenario), or more interestingly, be reached at $T \simeq 210$ MeV, as in the LO ChPT result (the second scenario). In both scenarios, we see no sign of violation of the universal minimum bound for η/s below T_c. But to really make sure the bound is valid from 120 MeV to T_c, a lattice computation as was performed to gluon plasma above T_c [11] is needed. In the second scenario, assuming the bound is valid for QCD, then either a phase transition or cross over should occur before the minimum bound is reached at $T \sim 200$ MeV. Also, in this scenario, it seems natural for η/s to stay close to the minimum bound around T_c as was recently found in heavy ion collisions.

In the second scenario, one might argue that the existence of phase transition is already known, otherwise we will not have spontaneous symmetry breaking and the corresponding Nambu-Goldstone boson theory at low temperature in the first place. Indeed, it is true in the case of QCD. For a spontaneous symmetry breaking theory, the general feature of η/s we see here seems generic.

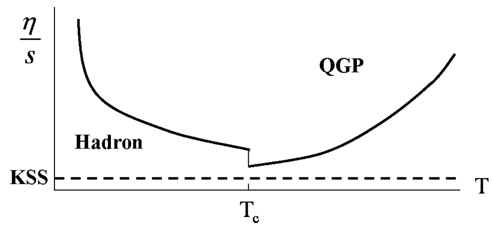

Fig. 4. A schematic of η/s behavior around the critical temperature.

At asymptotically high $T \gg T_c$ in the deconfinement phase, the weak coupling nature of QCD makes η/s get higher in comparison with that near T_c. On the other hand, at very low $T \ll T_c$ in the symmetry breaking phase, the Nambu-Goldstone bosons are weakly interaction at low temperature, thus η/s gets lower as T approaches T_c from below. A phase transition should occur before the extrapolated η/s curve coming from high T reaches the bound at T_1. Similarly, a phase transition should occur before the extrapolated η/s curve coming from low T reaches the bound at T_2. Thus the range of phase transition is $T_1 \leq T_c \leq T_2$, see Fig. 4. There might be a discontinuity at T_c due to the change of effective degrees of freedom. Some relations between T_c and the η/s bound are also discussed in Refs. [37, 38].

3.4 Large N_f and N_c limit

It is interesting to note that the degeneracy factor g_π drops out of η while the entropy s is proportional to g_π, as shown in Eqs. (26-28) and (29), respectively. This suggests the η/s bound might be violated if a system has a large particle degeneracy factor [2]. For QCD, large g_π can be obtained by having a large number of quark flavors N_f with $g_\pi \sim N_f^2$. However, the existence of confinement demands that the number of colors N_c should be of order N_f to have a negative QCD beta function. After using $f_\pi \propto \sqrt{N_c}$, the combined N_c and N_f scaling of η/s is

$$\frac{\eta}{s} \propto \frac{f_\pi^4}{g_\pi T^4} \propto \frac{N_c^2}{N_f^2} \ , \tag{31}$$

which is of order one. Thus QCD with large N_c and N_f can still be consistent with the η/s bound below T_c.

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